# EFFECT OF COUNTERFLOW MIXED CONVECTION ON THE HEAT TRANSFER IN VERTICAL LIQUID-METAL HEAT EXCHANGERS

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The results are shown here of an experimental and analytical study concerning the heat transfer between sodium and sodium–potassium alloy in counterflow heat exchangers with counterflow mixed convection in one of the ducts. It is shown that, at low Pe numbers, the heat transfer is determined by the effect of longitudinal heat overruns in the liquid.

Counterflow mixed convection of heat is observed in a liquid heated while moving downward or cooled while moving upward through ducts of large cross sections. This mode of heat transfer is characterized by the presence of a large turbulence component $\lambda_{TX}$ in the longitudinal thermal conductivity of the liquid – a result of the positive vertical gradient of liquid density and, as a consequence, of large heat overruns along the height (according to indirect estimates based on the data in [2], $\Lambda = \lambda_{TX}/\lambda$ may be equal to 10 or more). This makes the true "liquid-to-liquid" temperature drop $\Delta t$ in the heat exchanger much smaller than $\Delta t_p$ calculated outside the heat transfer zone. The effect of longitudinal heat overruns in a heat exchanger has been studied in [1, 2] for the case $\lambda_{TX} = 0$. The equations describing the change in the mean temperature of the liquid in the ducts at $W_1 = W_2$ are then:

$$t_1'' - \frac{\mathrm{Pe}}{\varkappa} t_1' + \frac{4\mathrm{Nu}_k}{\varkappa}(t_2 - t_1) = 0, \tag{1}$$

$$t_2'' + \mathrm{Pe}\, t_2' - 4\mathrm{Nu}_k (t_2 - t_1) = 0. \tag{2}$$

The boundary conditions are

$$\begin{aligned} X = 0 \quad & \text{a) } t_1 = \frac{\varkappa}{\mathrm{Pe}} t_1', \quad \text{b) } t_2' = 0; \\ X = L \quad & \text{c) } t_1' = 0, \quad \text{d) } t_2 = t_0 - \frac{t_2'}{\mathrm{Pe}}. \end{aligned} \tag{3}$$

Equations (1) and (2) apply where the wall across which heat transfer occurs has a negligibly small longitudinal component of thermal conductivity and the geometrical dimensions of the ducts are constant over their active lengths.

When $\Lambda = \text{const} \neq 0$ in one of the heat exchanger ducts (e.g., in the one denoted by subscript 2), then the respective equations become

$$t_1'' - \frac{\mathrm{Pe}}{\varkappa} t_1' + \frac{4\mathrm{Nu}_k}{\varkappa}(t_2 - t_1) = 0, \tag{4}$$

$$t_2'' + \frac{\mathrm{Pe}}{1+\Lambda} t_2' - \frac{4\mathrm{Nu}_k}{1+\Lambda}(t_2 - t_1) = 0. \tag{5}$$

Obviously, these equations will be identical to Eqs. (1), (2) if in the latter ones Pe, $\mathrm{Nu}_k$, and $\varkappa$ are replaced by $\mathrm{Pe}/(1+\Lambda)$, $\mathrm{Nu}_k/(1+\Lambda)$, and $\varkappa/(1+\Lambda)$ respectively.

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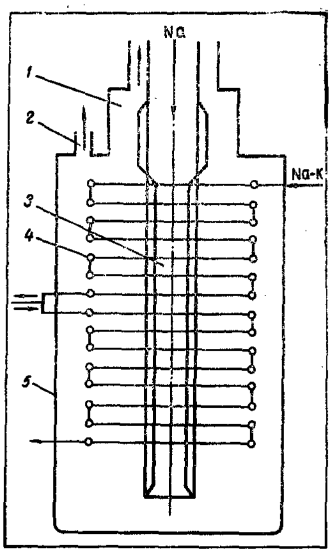

Fig. 1

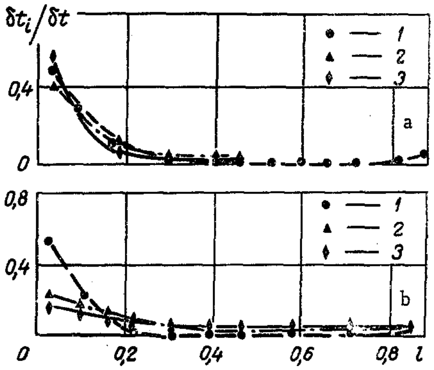

Fig. 2

Fig. 1. Annular test duct with a coil of tubing: 1) main outlet for sodium, 2) bypass outlet for sodium, 3) dropping tube with air for insulation, 4) coiled tubing, 5) housing.

Fig. 2. Operating effectiveness of coil turns: a) as a function of length, at Pe = 4.5 (1) full coil, 2) upper part, 3) lower part); b) as a function of Pe, at full length (1) Pe = 4.5, 2) Pe = 12.5, 3) Pe = 16.5). Length$l$,) (m).

We will consider the case where $\varkappa/(1 + \Lambda) \approx 0$ and Eq. (4) is

$$t_1' - \frac{4\mathrm{Nu}_k}{\mathrm{Pe}}(t_2 - t_1) = 0, \tag{6}$$

while boundary condition (3c) becomes meaningless. Solving (5) and (6) simultaneously, we obtain

$$t_1 = \frac{t_0}{H_0}\left[1 - \exp\beta X + \left(1 + \frac{\mathrm{Pe}}{4\mathrm{Nu}_k}\beta\right)\beta X\right], \tag{7}$$

$$t_2 = \frac{t_0}{H_0}\left[1 - \left(1 + \frac{\mathrm{Pe}}{4\mathrm{Nu}_k}\beta\right)\exp\beta X + \left(1 + \frac{\mathrm{Pe}}{4\mathrm{Nu}_k}\beta\right)\left(\frac{\mathrm{Pe}}{4\mathrm{Nu}_k} + X\right)\beta\right], \tag{8}$$

$$H_0 = \beta\left(1 + \beta\frac{\mathrm{Pe}}{4\mathrm{Nu}_k}\right)\left[\frac{\mathrm{Pe}}{4\mathrm{Nu}_k} + \frac{1+\Lambda}{\mathrm{Pe}} + L - \left(\frac{1}{\beta} + \frac{1+\Lambda}{\mathrm{Pe}}\right)\exp\beta L\right], \tag{9}$$

$$\beta = -\left[\frac{\mathrm{Pe}}{1+\Lambda} + \frac{4\mathrm{Nu}_k}{\mathrm{Pe}}\right].$$

Let $\pi = 4\mathrm{Nu}_k/\mathrm{Pe}^2(1 + \Lambda)$. Then

$$\Delta t = \frac{t_0}{H_0}\left(1 + \frac{1}{\pi}\right)\left(\frac{1}{\pi} + \exp\beta X\right), \tag{10}$$

$$\Delta t_\mathrm{p} = t_2|_{X=0} = \frac{t_0}{H_0}\left(\frac{1+\pi}{\pi}\right)^2. \tag{11}$$

There are two ways of calculating the quantity of heat Q transferred in the heat exchanger: $Q = K\Delta t$ and $Q = K_\mathrm{p}\Delta t_\mathrm{p}$. In the second case one tentatively includes the effect of longitudinal heat overruns in the heat transfer coefficient while the temperature drop is assumed to remain without change. Then

$$\mathrm{Nu}_\mathrm{cr} = \frac{K_\mathrm{p}d_2}{\lambda_2} = \frac{\overline{\Delta t}}{\Delta t_\mathrm{p}}\mathrm{Nu}_k = \frac{\mathrm{Nu}_k}{1+\pi} + \frac{\pi^2}{(1+\pi)^2}\cdot\frac{(1-\exp\beta L)}{4L}\mathrm{Pe}. \tag{12}$$

It is evident that the first term in (12) becomes equal to $\mathrm{Nu}_{\mathrm{cr}}$ at $L \to \infty$. At low Pe numbers and large values of $\Lambda$, $\pi \gg 1$ and

$$\mathrm{Nu}_{\mathrm{cr}} = \frac{\mathrm{Pe}^2}{4(1+\Lambda)} + \frac{(1-\exp\beta L)}{4L}\,\mathrm{Pe}. \tag{13}$$

On the basis of relations (7)-(13) one can draw certain conclusions about the heat transfer at large values of $\Lambda$. First of all, the true temperature drop (10) is very small in that part of the heat exchanger where $\beta X \to 0$, i.e., where the heat transfer occurs essentially at the end section (of the duct with $\Lambda > 0$) and the length of active end section decreases as the Pe number decreases. Secondly, the value of $\mathrm{Nu}_{\mathrm{cr}}$ and thus the actual quantity of heat transferred depend very little on the true value of the heat transfer coefficient K. This means that the form of the equation for $\mathrm{Nu}_{\mathrm{cr}}$ will, at low Pe numbers, be common for a large class of heat exchangers with ducts of various shapes. To this class will also belong heat exchangers whose geometrical parameters are either constant or periodically variable in lengthwise direction.

Let us analyze the individual quantities which enter into Eq. (13) for the general case. For this, we write the system of heat balance equations for a heat exchanger whose heat transferring surface and whose duct cross sections have arbitrary shapes, considering also that $F_1\lambda_1 \ll F_2(\lambda_2 + \lambda_{\mathrm{Tx}})$,

$$t_2'' + \frac{Wd_{\mathrm{T}}}{F_2\lambda_2(1+\Lambda)}\,t_2' - \frac{KS}{l}\cdot\frac{d_{\mathrm{T}}^2}{F_2\lambda_2(1+\Lambda)}\,(t_2-t_1) = 0,$$

$$t_1' - \frac{KS}{l}\cdot\frac{d_{\mathrm{T}}}{W}\,(t_2-t_1) = 0.$$

This system transforms into system (5), (6) if we substitute

$$\frac{Wd_{\mathrm{T}}}{F_2\lambda_2} = \mathrm{Pe},\quad \frac{KS}{l}\cdot\frac{d_{\mathrm{T}}^2}{4F_2\lambda_2} = \mathrm{Nu}_k. \tag{14}$$

According to (13),

$$\mathrm{Nu}_{\mathrm{cr}} = \frac{K_{\mathrm{p}}S}{l}\cdot\frac{d_{\mathrm{T}}}{4F_2\lambda_2}. \tag{14a}$$

Thus, when $\pi \gg 1$, in the general case the quantity $\mathrm{Nu}_{\mathrm{cr}}$ (14a) can be expressed in terms of Pe (14) and L coordinates.

This analysis leads to still another conclusion. If Eq. (13) is made explicit with the aid of (14) and (14a), we will obtain when $\exp\beta L \approx 0$:

$$K_{\mathrm{p}}S = \frac{W^2 l}{F_2\lambda_2(1+\Lambda)} + W. \tag{15}$$

The quantity of heat transferred in the heat exchanger is

$$Q = K_{\mathrm{p}}S\,\Delta t_{\mathrm{p}}.$$

It is then evident from (15) that the quantity of transferred heat will increase as a layer of impurities precipitates on the heat exchanger surfaces, since the role of a sediment when $\pi \gg 1$ is solely to reduce $F_2$ and to the same extent $\Lambda$. Inasmuch as the quantities Q and K in the experiments are determined from the equalities

$$Q = W\theta,\quad K_{\mathrm{p}} = \frac{W\theta}{S_0\Delta t_{\mathrm{p}}}$$

($S_0$ is the original surface area), so $K_{\mathrm{p}}$ will obviously increase.

As Pe increases while $\mathrm{Nu}_{\mathrm{k}}$ and $\Lambda$ decrease, $\pi$ decreases and the condition $\pi \gg 1$, for which the foregoing conclusions were made, becomes less valid.

The very quantities which determine $\pi$ depend on many factors. Thus, $\Lambda$ depends on Pe, $l$, $d_{\mathrm{T}}$, $\theta$, and on the design of the heat transfer zone (coil, fins, etc.), while $\mathrm{Nu}_{\mathrm{k}}$ depends on the thermal resistance of the walls and the oxide scale as well as on the heat transfer in both ducts. Therefore, apparently, the effects of the various parameters on the heat transfer is generally very complex. Qualitatively, however, they are quite accurately assessed on the basis of the foregoing analysis.

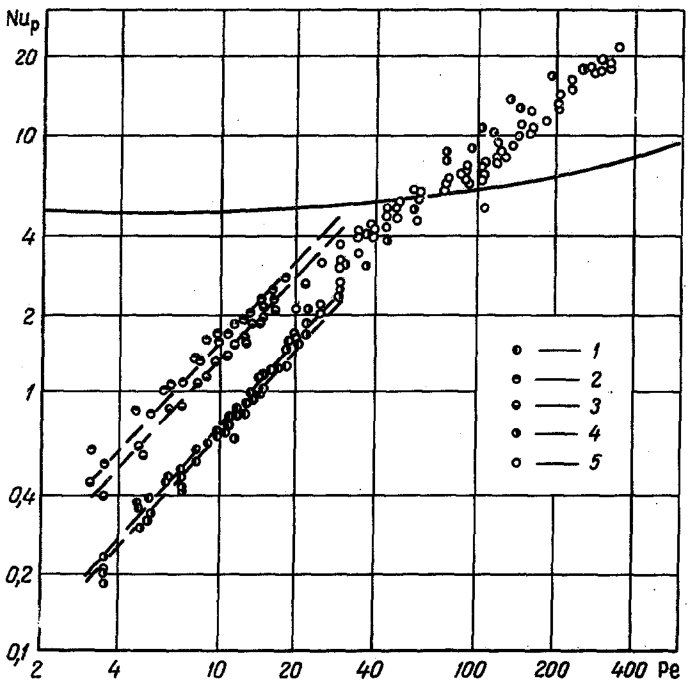

Fig. 3. Heat transfer with sodium during counterflow mixed convection: 1-3 annular duct with a coil (L = 4.74 (1), 2.21 (2), and 2.53 (3)), 4, 5 tubing ($d_2$ = 135 mm (4) and 68 mm (5)). Solid curve represents equation $Nu = 5 + 0.025Pe^{0.8}$, dashed curve represents equation $Nu_p = 0.27Pe^{1.1}/L$.

Experiments were performed with a "sodium/Na–K" heat exchanger shown in Fig. 1. Its main feature was the provision for measuring the temperature rise of the sodium–potassium alloy in each turn of the coil and, therefore, the distribution of heat transfer intensities as well as the mean temperatures of the alloy along the duct.

The Na–K ducts were arranged so that the sodium could be cooled by the entire coil (15 turns) as well as by its upper part (7 turns) or lower part (8 turns) only. This made it possible to find out how the heat transfer process is affected by the active length of the exchanger and by its inlet conditions.

In these experiments we measured the sodium temperatures at the inlet and at the outlet of the annular duct, and Na–K temperatures at the inlet to and at each turn of the coil, the flow rates of sodium and Na–K, and the uncompensated heat losses through the outer surfaces covered with calorimeters and compensating heaters.

The groups of tests were performed: the first one to determine the effect of Pe, L, and $\Delta t_p$ on the heat transfer with pure sodium (oxygen content not exceeding $5 \cdot 10^{-4}\%$ by weight) at $W_1 = W_2$, the second one to study the variation of $K_p$ during precipitation of sodium oxide in the test model at a constant flow rate of the heat carrier.

The values obtained for $K_p$ under pure conditions have shown – as have those obtained in experiments with concentric heat exchangers [2] – a tendency to follow the relation $K_p \sim W$, but they also show a divergent pattern with respect to L. The larger the number of active turns (larger L) is, the lower is the value of $K_p$.

Furthermore, from the temperature distributions in the Na–K alloy along the coil turns one can see that most of the heat transfer occurs through the turns near the end of the sodium duct. The curves in Fig. 2a represent the ratio of temperature rise per turn to temperature rise in the alloy over the entire coil length. It is evident here that, with the same Pe number, the end portion (with reference to the sodium) of the coil operates at the same intensity regardless of the active number of turns (length of coil). When the Pe number becomes higher, as can be seen in Fig. 2b, the "active" turns region widens and the thermal flux tends toward equilibrium over the entire length.

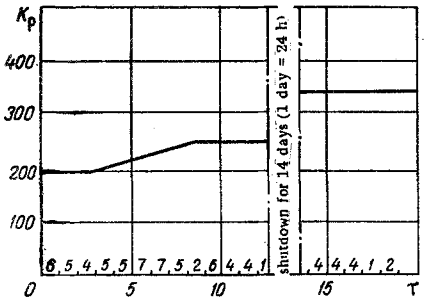

Fig. 4. Change in the calculated value of the heat transfer coefficient plotted against time during the precipitation of oxide (the numbers under the curve indicate the numbers of readings taken during a 24 h period). $K_p (W/m^2 \cdot {}^\circ C)$, $\tau$ (days).

Qualitatively, these facts agree with the results of analysis. Therefore, the data on heat transfer should also be generalized qualitatively on the basis of that analysis, i.e., by taking into account relations (14) and (14a).

In order to compute $Nu_{cr}$, Pe, and L, one must know S, $F_2$, and $d_T$. S denotes the area of the entire coil surface: $S = \pi d_0 \cdot \pi d_T n$. The presence of a coil makes the quantities $F_2$ and $d_T$ vary periodically. It appears that the method of calculating $F_2$ and $d_T$ is of little importance in the processing of experimental data. Indeed, according to (15), the value of $K_p$ does not depend on $d_T$ and very little on the total longitudinal thermal conductivity of the duct $F_2\lambda_2(1 + \Lambda)$. Since the value of $\Lambda$ is indeterminate, it is entirely permissible to disregard the coil in calculating $F_2$ and $d_T$:

$$F_2 = \frac{\pi}{4}(D_1^2 - d_1^2), \quad d_T = D_1 - d_1.$$

Bearing this in mind and according to (14a), we have

$$Nu_{cr} = \frac{K_p d_T}{\lambda_2} \cdot \frac{\pi d_0 n}{l} \cdot \frac{d_T}{D_1 + d_1}.$$

Here $d_0$ is the coil diameter, $d_T$ is the tube diameter, n is the number of turns, $D_1$ is the outside diameter of the annular duct, and $d_1$ is the inside diameter of the annular duct.

All data, including those in [2], have been processed in $Nu_p$(Pe) coordinates, as shown in Fig. 3, where

$$Nu_p = Nu_{cr} \frac{\alpha_{2p}}{K_p}, \quad \alpha_{2p} = \frac{\alpha_2 \overline{\Delta t_2}}{\Delta t_{2p}}.$$

This method of data processing has been necessary for the purpose of generalizing data which represent the heat transfer in the particular duct studied in our experiment. This can be done when in the relation

$$\frac{1}{K_p} = \frac{1}{\alpha_{2p}} + \rho_{wp} + \rho_{1p},$$

$1/\alpha_{2p}$ is much greater than $\rho_{wp}$ and $\rho_{1p}$. Here $\rho_{ip} = \rho_i(\Delta t_{ip}/\overline{\Delta t_i}) = \rho_i(1 + \delta t_i/\overline{\Delta t_i}) = \rho_i + \delta t_i/\overline{q}$ are the calculated (apparent) thermal resistances of the wall and of the annular clearance (the coil), $\delta t_i$ is the fraction by which the total temperature drop across the heat exchanger is reduced as a result of longitudinal heat overruns at the respective element. Thus, for the heat transferring surface of the "tube in a tube" heat exchanger we have

$$\rho_{wp} = \rho_w + \frac{F_w \lambda_w \partial t_w/\partial x}{W^2 \partial t_1/\partial x \, \frac{1}{\Pi}} \approx \rho_w + \frac{\varphi}{Pe^2} \cdot \frac{4d}{\lambda_2}.$$

By analogy, for the annular clearance:

$$\rho_{1p} = \rho_1 + \frac{\varkappa}{Pe^2} \cdot \frac{4d}{\lambda_2}.$$

As was to be expected, the test points for $Nu_p$ at low Pe numbers follow the qualitative predictions based on the analysis. Within the ranges $3 < Pe < 30$ and $2 < L < 5$ the data for all known heat exchangers fit sufficiently well into the formula:

$$Nu_p = 0.27 \frac{Pe^{1.1}}{L}. \qquad (16)$$

For simplicity, the term corresponding to $Nu_p/L \to \infty$ [see (12)] is not included here. The contribution of this term is, however, reflected in the coefficient and the exponent of Pe, which are here 0.27 and 1.1 respectively instead of 0.25 and 1.0 as in (12). The condition $\pi \gg 1$ is not maintained when $Pe > 30$, which

TABLE 1

| Part | I | II | III | IV | V |
|---|---|---|---|---|---|
| Quantity of oxide, g | 2360 | 2670 | 962 | 266 | 726 |
| % quantity found | 33.8 | 38.2 | 13.8 | 3.8 | 10.4 |

makes a description of the heat transfer process much more complicated. At the peak values of Pe the heat transfer process cannot become stable while the transverse turbulent heat transfer intensifies as a result of the longitudinal heat transfer, and this causes the Nu numbers to be high.

In order to determine the effect which the precipitation of oxides in the test part has on the heat transfer, we studied the operation at Pe = 7 and $\Delta t_p = 40$°C. Oxygen was injected into the system in the form of sodium peroxide in 70 g doses through a valve in the pump tank. Initially the sodium peroxide powder produced a brief fluctuation in the sodium flow rate, which then quickly stabilized. After 3 kg of sodium peroxide had been poured in, the hydraulic resistance of the tested heat exchanger section increased so much that the sodium temperature at the outlet had to be increased in order to bring the flow rate of sodium to its prescribed level (the flow rate of Na–K in the coil was reduced).

The sequence of adding sodium peroxide and the resulting change in the calculated heat transfer coefficient is shown plotted against time in Fig. 4. Only 5.3 kg sodium peroxide was added, which is equivalent to 8.5 kg or approximately 9 $dm^3$ sodium oxide (the acceptable volume concentration of oxide is approximately 40%). It is evident that, as oxide precipitated in the test section of the heat exchanger, the calculated value of the heat transfer coefficient increased. The stepwise increase of $K_p$ after shutdown had, apparently, to do with the drift of oxide from the outlet section of the apparatus into the heat transfer zone. Such a drift could occur, because after the shutdown it was not possible to establish the specific operating conditions and because for 72 h there was no Na–K alloy admitted, or it was let into a lower coil section only and at a low flow rate. The sodium temperature at the outlet from the annular duct fluctuated between 130 and 300°C. The values of $K_p$ were not determined at that time. The tendency for $K_p$ to increase as the oxide layer on the heat exchanger built up can be explained qualitatively on the strength of the earlier analysis (see Eq. (15)).

After completion of the experiment, the entire structure was cut into five parts for an oxygen content analysis. The parts were arranged as follows: the entire top down to the first coil turn (part I), turns 1-2-3 (part II), turns 4-5-6-7 (part III), turns 8-9-10-11 (part IV), and turns 12-13-14-15 with the bottom (part V). A chemical analysis yielded 6984 g of oxide, i.e., 82.2% of the amount introduced.

The distribution of oxide content among the various parts was as shown in Table 1.

It is evident from the results of the chemical analysis that a large portion of the oxide precipitated in the zone of most intensive heat transfer. A quite considerable portion of the oxide precipitated above, in the isothermal zone. The high oxide content in this region explains why the duct became clogged up already during the first half of the period of feeding in the sodium peroxide.

## NOTATION

| | |
|---|---|
| $t$ | is the mean-calorimetric temperature of the liquid, °C; |
| $t_0$ | is the temperature at the main heat exchanger inlet, °C; |
| $t_i^{(j)} = \partial^{(j)} t_i / \partial x^{(j)}$, °C; | |
| $\theta$ | is the temperature rise of the liquid in the heat exchanger duct, °C; |
| $x$ | is the longitudinal coordinate, m; |
| $l$ | is the length of heat exchanger section, m; |
| $d$ | is the duct diameter, m; |
| $X = x/d$; | |
| $L = l/d$; | |
| $\Pi$ | is the circumference of heat exchanger, m; |
| $\varkappa = F_1\lambda_1/F_2\lambda_2$; | |
| $\varphi = F_w\lambda_w/F_2\lambda_2$; | |
| $F$ | is the cross section area, $m^2$; |
| $S$ | is the surface area of heat exchanger, $m^2$; |
| $w$ | is the velocity, m/sec; |
| $w = F_w C_p \gamma$; | |

| | |
|---|---|
| $C_p$ | is the specific heat, J/kg · °C; |
| $\gamma$ | is the density, $kg/m^3$; |
| $\lambda$ | is the thermal conductivity, W/m · °C; |
| $a$ | is the thermal diffusivity, $m^2/sec$; |
| $\alpha$ | is the heat transfer; |
| k | is the heat exchange coefficient, $W/m^2 \cdot °C$; |

$Pe = wd_T/a$;

$Nu_k = Kd_2/\lambda_2$.

Subscripts

| | |
|---|---|
| 1 | is the auxiliary heat exchanger duct; |
| 2 | is the main duct; |
| w | is the heat transferring wall; |
| p | is the predicted value (calculated from the temperature drop at some distance from the heat exchanger). |

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